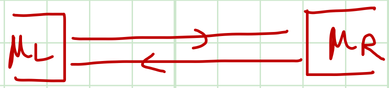
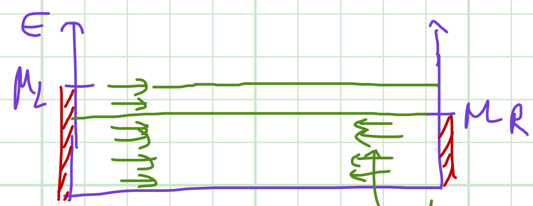
Quantum Hall Edge State conductance

Last time: conductivity of a 1D quantum channel





these are on the far edge

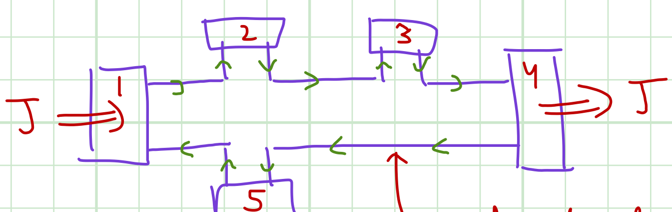
$$J = e\int_{\mu_R}^{\mu_L} \underbrace{\frac{1}{\hbar}\frac{\partial E_k}{\partial k}}_{\Uparrow \text{ velocity}} \frac{dk}{2\pi} = \frac{e^2 V}{h} \quad \text{where} \quad \mu_L - \mu_R = eV$$

$\sigma = \frac{J}{V} = e^2/h \Leftarrow$ per channel!

Transport in QHE System:

The main feature of QHE that we will use is the fact that the edge states are "chiral" they go around the sample in only one direction.

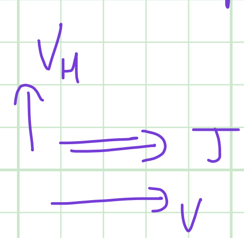
Consider a Hall bar



chiral edge states $\Rightarrow$ carry current

conventional metal contacts $\Rightarrow$ equipotential surfaces

good setup to measure resistivity



$\Rightarrow$ easy to measure voltage // and $\perp$ to current

contacts 1 and 4 are biased so that current 1 $\Rightarrow$ 4 is J.

contacts 2, 3, 5 are floating $\rightarrow$ we use these to measure voltage, but no current flows through them.

Question what is V_{25} and V_{23}? [hence R_{xy} and R_{xx}]

consider the current J_{12} [ie current ① ⇒ ②]

Since the edge state is chiral, all states with $\varepsilon < \mu_1$ will be occupied and hence will contribute to the current.

$$J_{12} = e\int_{W}^{\mu_1} \frac{dk}{2\pi}\left[\frac{1}{\hbar}\frac{d\varepsilon}{dk}\right] = \frac{e}{h}(\mu_1 - W) = \frac{e^2}{h}(V_1 - W/e) = \sigma_0 (V_1 - W/e)$$

bottom edge of the band

Following similar logic, $J_{23} = \sigma_0(V_2 - W/e)$, $J_{34} = \sigma_0(V_3 - W/e)$, $J_{45} = \sigma_0(V_4 - W/e)$ and $J_{51} = \sigma_0(V_5 - W/e)$

Current conservation demands

(1) $\left.\begin{aligned} J_{12} &= J_{23} \\ J_{23} &= J_{34} \\ J_{45} &= J_{51} \end{aligned}\right\}$ no current through floating contact

(2) $J_{12} - J_{51} = J$ } total current injected into the device

Putting (1) + (2) together, we find a set of equations for the voltages:

$V_1 = V_2$

$V_2 = V_3$

$V_4 = V_5$

$\sigma_0(V_1 - V_5) = J$

⇒ We have 5 unknown voltages but only 4 equations!

⇒ Voltages are relative we can fix one of them e.g. $V_1 = 0$.

Resistivities: $V_2 - V_3 = 0 \Rightarrow R_{xx} = \frac{V_2 - V_3}{J} = 0$ (✓)

$V_2 - V_5 = V_1 - V_5 = J/\sigma_0 \Rightarrow R_{xy} = \frac{V_1 - V_5}{J} = \frac{J/\sigma_0}{J} = \frac{1}{\sigma_0}$ (✓)

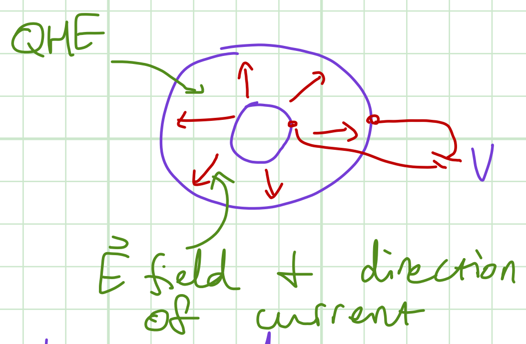
How to measure conductivity?

⇒ need to force a voltage + detect current // + ⊥ to it.

⇒ this is harder experimentally → std practice is to measure R_{xx} + R_{xy} and then invert Resistivity matrix.

⇒ We can see that σ_{xx} is indeed zero from the

Corbino geometry:



Corbino disk is a disk with a hole in the middle. We can measure σ_{xx} by applying a voltage between the inner + outer edges of the Corbino disk + measuring the current. In the QHE state the bulk of the disk is gapped + hence $J=0$.

$\sigma_{xx} = J/V = 0$ (✓)

Defects in crystals:

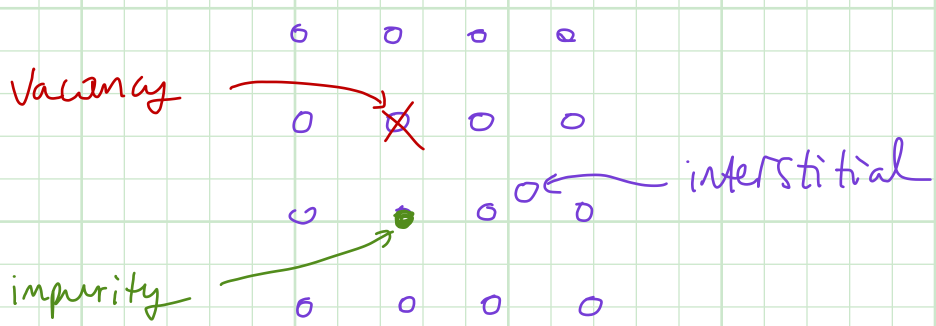
$\Rightarrow$ scattering centers that are important for e^- transport

($e^- - e^-$ and e^--ph scattering can also be important for transport)

$\Rightarrow$ important for plasticity + elasticity of materials.

Zoology of defects:

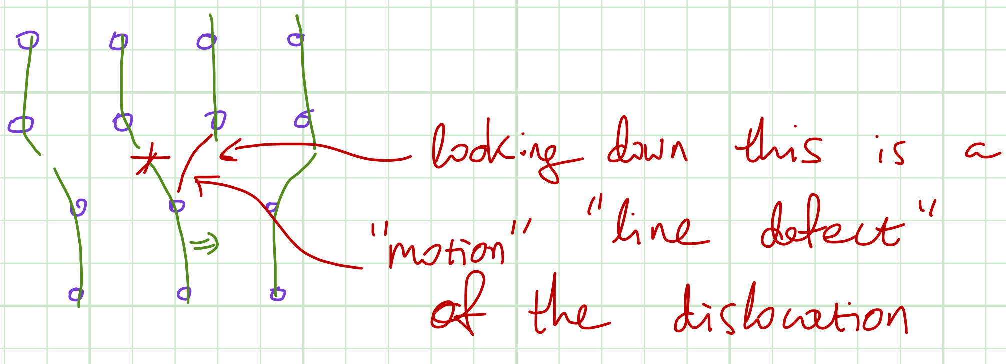
(1) Point defects:

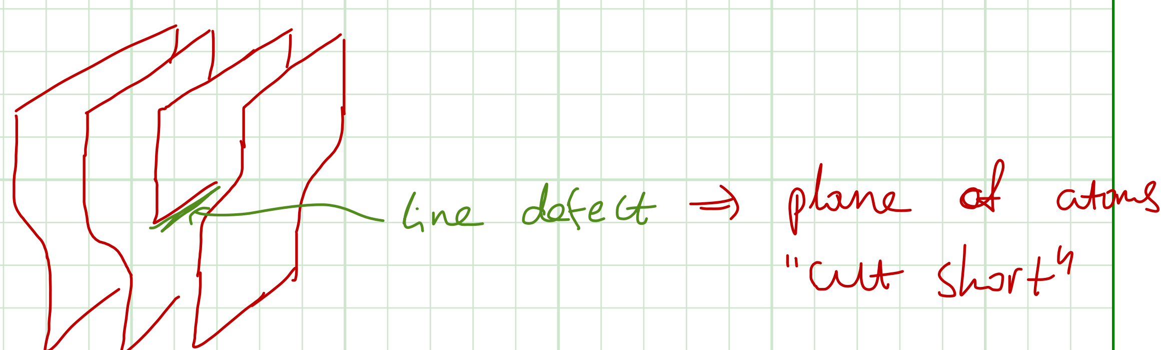


(2) line defects:

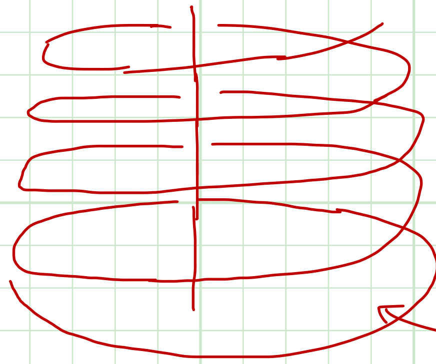
Edge Dislocations

$\Rightarrow$ these are important for plasticity + elasticity





Screw Dislocations

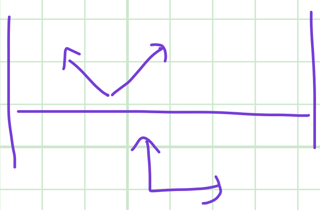


planes of atoms do not "line up" just like levels of parking lot.

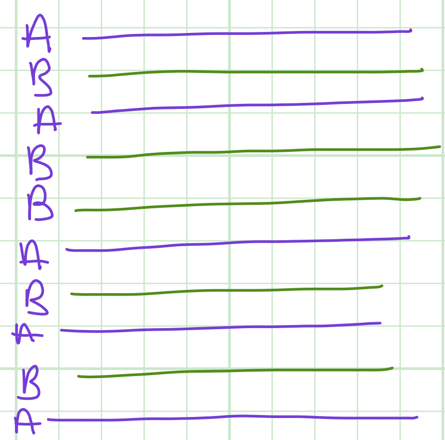
⇒ Important for "skew" scattering which is a source of anomalous Hall effect (i.e. Hall effect with no B-field)

(3) planar defects

⇒ aka grain boundaries



or



Linear Response: A classical model of polarization

DS: Relation between $\vec{E}$ and $\vec{P}$

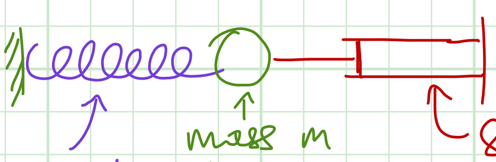
$$\vec{P} = \epsilon_0 \chi \vec{E}$$

(1) This is linear response: amount of response (amount of polarization) is proportional to the amount of driving (amount of E-field)

(2) So far relation between $\vec{P}$ + $\vec{E}$ was assumed, but how does one compute it?

We need a model!

Most crude, but still useful, model:
"classical charged particle on a spring model"



(1) What is the equation of motion for our model? [focus on 1D]

$$m\ddot{x} = \underset{①}{-Kx} \underset{②}{- m\Gamma\dot{x}} + \underset{③}{qE e^{i\omega t}}$$

① ⇒ restoring force ⇒ $K \to m\omega_0^2$ $\left[\omega_0 = \sqrt{K/m}\right]$
② ⇒ Damping force
③ ⇒ Driving force

(2) What is the origin of the model

Model makes most sense in an insulator where $\bar{e}$ are tied to nuclei.

⇒ restoring force – attraction of $\bar{e}$ to nucleus
⇒ Damping = coupling to phonons

(3) Solution of the model

$\Rightarrow$ Make a guess that $x(t) = x_0 e^{i(\omega t + \delta)}$

$\Rightarrow$ charge has to ossilate at the drive freq. ω, but there can be a phase shift δ.

$\Rightarrow$ Substitute guess into EOM:

$$\underbrace{-m\omega^2 x_0 e^{i(\cancel{\omega t}+\delta)}}_{m\ddot{x}} = \underbrace{-m\omega_0^2 x_0 e^{i(\cancel{\omega t}+\delta)}}_{-m\omega_0^2 x} \underbrace{- i m \Gamma \omega e^{i(\cancel{\omega t}+\delta)}}_{-m\Gamma\dot{x}} + \underbrace{q E_0 \cancel{e^{i\omega t}}}_{qE(t)}$$

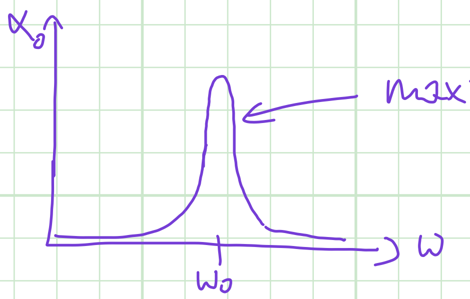
$$x_0 e^{i\delta}\left[m\omega_0^2 - m\omega^2 + i m \Gamma \omega\right] = q E_0$$

$$x_0 = \frac{qE_0/m}{\sqrt{(\omega_0^2-\omega^2) + \Gamma^2\omega^2}}$$

$$e^{i\delta} = \frac{qE_0}{x_0 m\left(\omega_0^2-\omega^2+i\Gamma\omega\right)} = \frac{\cancel{qE_0/m}\,\sqrt{(\omega_0^2-\omega^2)+\Gamma^2\omega^2}}{\cancel{qE_0/m}\,\left(\omega_0^2-\omega^2+i\Gamma\omega\right)}$$

$$\delta = \tan^{-1}\left[\frac{-\Gamma\omega}{\omega_0^2-\omega^2}\right]$$

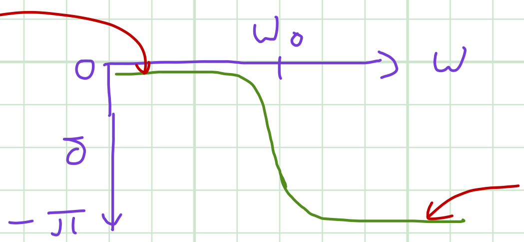
(4) Meaning of the Solution



maximum centered on ω_0

$\Rightarrow$ This is the natural oscillatation freq

low ω, charge follows E-field



high-freq. out-of-phase oscillations

(5) Susceptibility

N/V = density of charges

$$\chi \equiv P/\epsilon_0 E = \frac{\rho q x}{\epsilon_0 E} = \frac{\rho q}{\epsilon_0 E} x_0 e^{i\delta} = \frac{\rho q}{\epsilon_0 \cancel{E}} \frac{q\cancel{E_0}/m}{\cancel{\sqrt{(\omega_0^2-\omega^2)+\Gamma^2\omega^2}}} \frac{\cancel{\sqrt{(\omega_0^2-\omega^2)+\Gamma^2\omega^2}}}{\left(\omega_0^2-\omega^2+i\Gamma\omega\right)}$$

$$= \frac{\rho q^2}{m\epsilon_0}\left[\frac{1}{\omega_0^2-\omega^2+i\Gamma\omega}\right]$$

Propagation of light in medium – use of the susceptibility

Maxwell's wave equation in medium reads:

$$\frac{\partial D}{\partial t} = \nabla \times H \qquad \text{where} \qquad D \equiv \varepsilon_0 E + P = \varepsilon_0 (1+\chi) E$$

$$H = B/\mu_0 - M = B/\mu_0 \qquad [M=0]$$

$$\frac{\partial B}{\partial t} = -\nabla \times E$$

$$\Rightarrow \quad \frac{\partial^2}{\partial t^2} \mu_0 \varepsilon_0 (1+\chi) E = \nabla^2 E = \frac{(1+\chi)}{c^2} \frac{\partial^2 E}{\partial t^2}$$

writing $\chi = \chi_R + i\chi_I \qquad E = E_0 e^{i(kx - \omega t)}$

$$\frac{(1+\chi_R + i\chi_I)}{c^2} \omega^2 \cancel{E_0 e^{i(kx-\omega t)}} = k^2 \cancel{E_0 e^{i(kx-\omega t)}}$$

$$(k_R + i k_I)^2 = k_R^2 - k_I^2 + 2i k_R k_I = \frac{\omega^2}{c^2}(1 + \chi_R + i\chi_I)$$

$$k_R^2 - k_I^2 = \frac{\omega^2}{c^2}(1+\chi_R)$$

$$k_R k_I = \frac{\omega^2}{2c^2} \chi_I$$

$\Rightarrow$ If $\chi_I \neq 0 \Rightarrow k_I \neq 0 \Rightarrow$ decaying wave solution

$$E = E_0 \, e^{i(k_R x - \omega t)} \, e^{-k_I x}$$